

MATHEMATICS

(Three hours)

(Candidates are allowed additional 15 minutes for **only** reading the paper.

They must **NOT** start writing during this time.)

The Question Paper consists of **three** sections A, B and C.

Candidates are required to attempt all questions from **Section A** and all questions

EITHER from **Section B** **OR** **Section C**

Section A: Internal choice has been provided in three questions of four marks each and two questions of six marks each.

Section B: Internal choice has been provided in two questions of four marks each.

Section C: Internal choice has been provided in two questions of four marks each.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables and graph papers are provided.

SECTION A (80 Marks)

Question 1

[10x2]

- (i) Let $f: R \rightarrow R$ be a function defined by $f(x) = \frac{x-m}{x-n}$, where $m \neq n$. Then show that f is one-one but not onto.
 - (ii) Find the domain and range of the function $f(x) = [\text{Sin}x]$.
 - (iii) Find the square root of complex number $11 - 60i$.
 - (iv) For what value of k will the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ have one common root.
 - (v) In a ΔABC , show that $\sum(b + c) \cos A = 2s$ where $s = \frac{a+b+c}{2}$
 - (vi) Find the number of ways in which 6 men and 5 women can dine at a round table if no two women are to sit together.
 - (vii) Prove that $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{\sqrt{3}}{8}$
 - (viii) If two dice are thrown simultaneously, find the probability of getting a sum of 7 or 11.
 - (ix) Show that $\lim_{x \rightarrow 2} \frac{|x-2|}{x-2}$ does not exist.
 - (x) Find the point on the curve $y^2 = 4x$, the tangent at which is parallel to the straight line $y = 2x + 4$.
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Question 2 [4]

Draw the graph of the function $y = |x - 2| + |x - 3|$.

Question 3 [4]

Prove that $\cot A + \cot(60 + A) + \cot(120 + A) = 3 \cot 3A$.

OR

In a ΔABC prove that $b \cos C + c \cos B = a$

Question 4 [4]

Find the locus of a complex number, $Z = x + iy$, satisfying the relation $\left| \frac{z-3i}{z+3i} \right| \leq \sqrt{2}$.
Illustrate the locus of Z in the argand plane.

Question 5 [4]

Find the number of words which can be formed by taking four letters at a time from the word "COMBINATION".

OR

A committee of 7 members has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

- (i) exactly 3 girls
- (ii) at least 3 girls and
- (iii) at most three girls.

Question 6 [4]

Prove by the method of induction.

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots \text{upto } n \text{ terms} = \frac{n}{n+1} \text{ where } n \in N.$$

Question 7 [4]

Find the term independent of x and its value in the expansion of $\left(\sqrt{\frac{x}{3}} - \frac{\sqrt{3}}{2x} \right)^{12}$.

OR

Find the sum of the terms of the binomial expansion to infinity:

$$1 + \frac{2}{4} + \frac{2.5}{4.8} + \frac{2.5.8}{4.8.12} + \dots \text{to } \infty.$$

Question 8 [4]

Differentiate from first principle: $f(x) = \sqrt{3x + 4}$.

Question 9 [4]

Reduce the equation $x + y + \sqrt{2} = 0$ to the normal form ($x \cos \alpha + y \sin \alpha = p$) and find the values of p and α .

Question 10 [4]

Write the equation of the circle having radius 5 and tangent as the line $3x - 4y + 5 = 0$ at (1, 2).

Question 11 [6]

In a ΔABC prove that $\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$

Question 12 [6]

Find the n th term and deduce the sum to n terms of the series:
 $4 + 11 + 22 + 37 + 56 + \dots$

OR

If $(p+q)$ th term and $(p-q)$ th terms of G.P are a and b respectively, prove that p th term is \sqrt{ab} .

Question 13 [6]

If x is real, prove that the value of the expression $\frac{(x-1)(x+3)}{(x-2)(x+4)}$ cannot be between $\frac{4}{9}$ and 1.

OR

If x^p occurs in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$, prove that its coefficient is

$$\frac{(2n)!}{\left[\frac{1}{3}(4n-p)!\right] \left[\frac{1}{3}(2n+p)!\right]}$$

Question 14 [6]

Calculate the standard deviation of the following distribution:

Age	20-25	25-30	30-35	35-40	40-45	45-50
No. of persons	170	110	80	45	40	35

SECTION B (20 Marks)

Question 15

- (a) Find the focus and directrix of the conic represented by the equation $5x^2 = -12y$. [2]
- (b) Construct the truth table $(\sim p \wedge \sim q) \vee (p \wedge \sim q)$. [2]
- (c) Write the converse, contradiction and contrapositive of statement
“ If $x + 3 = 9$, then $x = 6$.” [2]

Question 16

Show that the point (1, 2, 3) is common to the lines which join A(4, 8, 12) to B(2, 4, 6) and C(3, 5, 4) to D(5, 8, 5).

OR

Calculate the Cosine of the angle A of the triangle with vertices A(1, -1, 2) B (6, 11, 2) and C(1, 2, 6).

Question 17

Find the equation of the hyperbola whose focus is (1, 1), the corresponding directrix $2x + y - 1 = 0$ and $e = \sqrt{3}$.

OR

Find the equation of tangents to the ellipse $4x^2 + 5y^2 = 20$ which are perpendicular to the line $3x + 2y - 5 = 0$.

Question 18

Show that the equation $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents a hyperbola. Find the lengths of axes and eccentricity.

SECTION C (20 Marks)

Question 19

- (i) Two sample sizes of 50 and 100 are given. The mean of these samples respectively are 56 and 50. Find the mean of size 150 by combining the two samples [2]
- (ii) Calculate P_{95} , for the following data : [4]

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	3	7	11	12	23	4

OR

Calculate Mode for the following data: [4]

C.I.	17-19	14-16	11-13	8-10	5-7	2-4
Frequency	12	4	8	16	11	4

Question 20

- (i) Find the covariance between X and Y when $\sum X = 50$, $\sum Y = -30$, and $\sum XY = 115$. [2]
- (ii) Calculate Spearman's Rank Correlation for the following data and interpret the result: [4]

Marks in Mathematics	36	48	27	36	29	30	36	39	42	48
Marks in Statistics	27	45	24	27	31	33	35	45	41	45

OR

Find Karl Pearson's Correlation Coefficient from the given data: [4]

x	21	24	26	29	32	43	25	30	35	37
y	120	123	125	128	131	142	124	129	134	136

Question 21

Find the consumer price index for 2007 on the basis of 2005 from the following data using weighted average of price relative method: [4]

Items	Food	Rent	Cloth	Fuel
Price in 2005(Rs)	200	100	150	50
Price in 2007(Rs)	280	200	120	100
Weighted	30	20	20	10

Question 22

Using the following data. Find out the trend using Quaterly moving average and plot them on graph: [4]

Year/ Quarter	Q ₁	Q ₂	Q ₃	Q ₄
1994	29	37	43	34
1995	90	42	55	43
1996	47	51	63	53